Foundation for Success

Unified International
Mathematics Olympiad

## UNIFIED INTERNATIONAL MATHEMATICS OLYMPIAD

## CLASS - 7 <br> Question Paper Code : UM9264

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## EXPLANATIONS

## MATHEMATICS - 1

1. (B) $(256)^{0.16} \times(256)^{0.09}=(256)^{0.16+0.09}$

$$
\begin{aligned}
& =\left(4^{4}\right)^{0.25} \\
& =4
\end{aligned}
$$

2. (C) Given $x+y=4 x y$

$$
\begin{aligned}
& \frac{x}{x y}+\frac{y}{x y}=\frac{4 x y}{x y} \\
& \frac{1}{x}+\frac{1}{y}=4
\end{aligned}
$$

3. (A) Let A's amount be ₹ $x$

Given ₹ $x+₹ x-16+3 x=₹ 129$.
$5 x=₹(129+16)=₹ 145$
$\therefore$ A's amount $=x=₹ \frac{145}{5}=₹ 29$
04. (C) $998^{2}-2 \times 998 \times 1002+1002^{2}$
$=996004-1999992+1004004$
= 2000008 - 1999992
= 16
05. (A) Radius of outer circle $=14 \mathrm{~cm}$

$\Rightarrow$ Circumference
$=2 \times \frac{22}{7} \times 14=88 \mathrm{~cm}$
Radius of inner circle
$=14-3.5=10.5 \mathrm{~cm}$
$\Rightarrow$ Circumference
$=2 \times \frac{22}{7} \times 10.5$
$=66 \mathrm{~cm}$
$\therefore$ Difference of circumferences
$=88-66 \mathrm{~cm}$
$=22 \mathrm{~cm}$
06. (D) Given

$$
\begin{aligned}
& \frac{27+x+31+x+98+x+105+x+164+x}{5}=101 \\
& 425+5 x=101 \times 5=505 \\
& 5 x=505-425=80 \\
& x=\frac{80}{5}=16 \\
& \therefore \text { Required mean }
\end{aligned}
$$

$=\frac{\binom{115+x+126-x+68-2 x}{+73+4 x+56+5 x}}{5}$
$=\frac{438+7 x}{5}=\frac{438+7 \times 16}{5}=\frac{438+112}{5}$
$=\frac{550}{5}=110$
07. (C) $\left[(43)^{3}+3 \times(43)^{2} \times 57+3 \times 43 \times(57)^{2}+\right.$ $\left.(57)^{3}\right]$
$=79507+171 \times 1849+129 \times 3249+$ 185193
$=79507+316179+419121+185193$
= 10,00,000
08. (A) Given $y-5^{\circ}+2 x+40^{\circ}=2 y-x$
$2 x+x+35^{\circ}=2 y-y$
$3 \times 5^{\circ}+35^{\circ}=y$
$y=35^{\circ}+15^{\circ}=50^{\circ}$
09. (A) $x^{2}-2 x-x(x-1)=x^{2}-2 x-x^{2}+x=-x$ $=-2022$
10. (C) $\therefore$ LCM $=5 \times 9 \times 7 \times 23 \times 5 \times 4=144900$

| 5 | 36225 | 28980 |
| ---: | :--- | ---: |
| 9 | 7245 | 5796 |
| 7 | 805 | 644 |
| 23 | 115 | 92 |
|  | 5 | 4 |

11. (D) Given $R=14 \mathrm{~cm} \& r=10.5 \mathrm{~cm}$


Difference between the area $=\pi R^{2}-\pi r^{2}$
$=\pi\left(R^{2}-r^{2}\right)$
$=\pi(R+r)(R-r)$
$=\frac{22}{7} \times(14+10.5) \mathrm{cm}(14-10.5) \mathrm{cm}$
$=\frac{22}{7} \times 24.5 \mathrm{~cm} \times 3.5 \mathrm{~cm}$
$=269.5 \mathrm{~cm}^{2}$
12. (B) $\ln \triangle \mathrm{ABC}, \angle \mathrm{B}=90^{\circ}$


$$
\begin{aligned}
& \therefore A C^{2}=A B^{2}+B C^{2} \\
& 58^{2}=40^{2}+B C^{2} \\
& 3364=1600+B C^{2} \\
& B C^{2}=3364-1600 \\
& B C^{2}=1764 \\
& B C^{2}=(42)^{2}
\end{aligned}
$$

Area of $\triangle A B C=\frac{1}{2} \times A C \times B D=\frac{1}{2} \times A B \times B C$
$=\frac{1}{2} \times 58 \mathrm{~cm} \times \mathrm{BD}=\frac{1}{2} \times 40 \times 42 \mathrm{~cm}^{2}$
$B D=\frac{1}{2} \times 40 \times 42 \times 2 \times \frac{1}{58}$
$=\frac{840}{29} \mathrm{~cm}=28 \frac{28}{29} \mathrm{~cm}$
13. (B) $\left(1+x+x^{2}+x^{3}+x^{4}+\right.$ $\qquad$ $\left.+x^{2020}\right)=$
$1+\left(x+x^{2}+x^{3}+x^{4}\right)+\left(x^{5}+x^{6}+x^{7}+x^{8}\right)+$
$\qquad$ $+\left(x^{2017}+x^{2018}+x^{2019}+x^{2020}\right.$
$=1+x\left(1+x+x^{2}+x^{3}\right)+x^{5}\left(1+x+x^{2}+x^{3}\right)$
$+$. $\qquad$ $+x^{2017}\left(1+x+x^{2}+x^{3}\right)$
$=1+x(0)+x^{5}(0)+\ldots \ldots+x^{2017}(0)$
$=1$
14. (C) Let the three consecutive numbers be $x$,
$x+1 \& x+2$
Given $2 x+3(x+1)+4(x+2)=245$
$2 x+3 x+3+4 x+8=245$
$9 x+11=245$
$9 x=245-11=234$
$x=\frac{234}{9}$
$\therefore$ The least number $=26$
15. (B) Given $l b=45 \frac{5}{16} \mathrm{~cm}^{2}$
$7 \frac{1}{4} \mathrm{~cm} \times \mathrm{b}=\frac{725}{16} \mathrm{~cm}^{2}$
$\frac{29}{4} \times b=\frac{725}{16} \mathrm{~cm}^{2}$
$b=\frac{725}{16} \times \frac{4}{29} \mathrm{~cm}$
$\frac{25}{4} \mathrm{~cm}=6 \frac{1}{4} \mathrm{~cm}$
$\therefore$ Perimeter $=2(l+b)=2\left(\frac{29}{4}+\frac{25}{4}\right) \mathrm{cm}$
$=2\left(\frac{29+25}{4}\right) \mathrm{cm}$
$=2 \times\left(\frac{54}{4}\right) \mathrm{cm}=27 \mathrm{~cm}$
16. (C) Given a:b=2:3=2x:3x

But $\angle \mathrm{a}+\angle \mathrm{b}=180^{\circ}$
$\therefore 2 x+3 x=180^{\circ}$
$5 x=180^{\circ}$
$x=\frac{180^{\circ}}{5}=36^{\circ}$
$\therefore \mathrm{a}=2 x=72^{\circ}$
$\mathrm{b}=3 x=108^{\circ}$
$\therefore \angle \mathrm{c}=\angle \mathrm{b}=108^{\circ}$
[corresponding angles]
$\therefore \angle \mathrm{d}=180^{\circ}-\angle \mathrm{c}=180^{\circ}-108^{\circ}=72^{\circ}$
$\therefore \angle \mathrm{c}-\angle \mathrm{d}=108^{\circ}-72^{\circ}=36^{\circ}$
17. (D) $1+2+3+4-5-6-7-8=10-26=-16$
$9+10+11+12-13-14-15-16$
$=42-58=-16$
Number of sets of 8 numbers up to 2016
$=\frac{2016}{8}=252$
$\therefore$ Sum of this 252 sets $=(252) \times(-16)$
$=-4032$
Required result $=-4032+2017+2108+$ 2019 + 2020-2021-2022-2023
$=-4032+8074-6066$
$=-2024$
18. (B) Second expression $=$ Sum - first expression
$=\left(x^{3}+x^{2}-3 x+2\right)-\left(x^{3}-x^{2}-6 x-8\right)$
$=x^{3}+x^{2}-3 x+2-x^{3}+x^{2}+6 x+8$
$=2 x^{2}+3 x+10$
19. (C) $2 x \mid 10 x^{2} y, \quad 25 x^{2} y^{2}, \quad 8 x y^{2} z^{3}$

| $5 x$ | $5 x y, 25 x y^{2}, 4 y^{2} z^{3}$ |
| ---: | ---: |
| $y$ | $y, 5 y^{2}, 4 y^{2} z^{3}$ |
| $y$ | $1,5 y, 4 y z^{3}$ |
|  | $1,5,4 z^{3}$ |

$\therefore \mathrm{LCM}=(2 x)(5 x)(y)(y)(5)\left(4 z^{3}\right)$
$=200 x^{2} y^{2} z^{3}$
20. (C) S.A.S. criterion is satisfied if two sides and an included angle of a triangle are equal to the two corresponding sides and included angle of the other
21. (C) $\mathrm{LHS}=-\frac{3}{10}-\frac{9}{10}+\frac{7}{15}-\frac{13}{15}+\frac{3}{20}-\frac{13}{20}$
$=\frac{-3-9}{10}+\frac{7-13}{15}+\frac{3-13}{20}$
$=\frac{-12}{10}-\frac{6}{15}-\frac{10}{20}$
$=\frac{-6}{5}-\frac{2}{5}-\frac{1}{2}$
$=\frac{-12-4-5}{10}=\frac{-21}{10}$
22. (A) $60^{\circ} \& y$ are vertically opposite angles which are equal
$\Rightarrow y=60^{\circ}$
In the triangle, $x+60^{\circ}+40^{\circ}=180^{\circ}$
(Angle sum property)
$\Rightarrow x+100^{\circ}=180^{\circ}$
$\Rightarrow x=180^{\circ}-100^{\circ}=80^{\circ}$
23. (D) Number of marbles Pankaj has $=96$

Number of marbles Arun has $=63$
Let the number of marbles that Arun should give Pankaj be ' $x$ '
Then according to the problem, $(96+x)$
$=2(63-x)$
$\Rightarrow 96+x=126-2 x$
$\Rightarrow 3 x=126-96$
$\Rightarrow 3 x=30 \Rightarrow x=10$
24. (B) The range of a given data is the difference between its highest and lowest values
From the given data, range
$=154-128=26$
25. (D) Given $\bigcirc+\bigcirc+\square=11$ and

$\therefore \bigcirc=11-8=3$
Given $\bigcirc+\triangle+\triangle=15$
$3+2 \triangle=15$
$2 \triangle=15-3$
$2 \triangle=12$
$\triangle=6$
26. (A) $S P=\frac{5}{4} C P$

Profit $=S P-C P=\frac{5 C P}{4}-C P$
$=\frac{5 C P-4 C P}{4}=\frac{C P}{4}$
$\therefore$ Profit percentage $=\frac{\text { Profit }}{C P} \times 100$

$$
\begin{aligned}
& \frac{C P}{4} \\
= & \frac{C P}{C P} \\
= & 25 \%
\end{aligned}
$$

27. (C) Here we equate the areas,
i.e., $A B \times D L=B C \times D M$
$\Rightarrow 18 \times \mathrm{DL}=12 \times 10$
$\Rightarrow D L=\frac{12 \times 10}{18}=\frac{20}{3}=6 \frac{2}{3}$
28. (D) Given $3 x-9^{\circ}+3 x+5 x+2^{\circ}=180^{\circ}$
$11 x-7^{\circ}=180^{\circ}$
$11 x=180^{\circ}+7^{\circ}=187^{\circ}$
$x=\frac{187^{\circ}}{11}=17^{\circ}$
$\therefore 3 x-9^{\circ}=3 \times 17^{\circ}-9=51^{\circ}-9^{\circ}=42^{\circ}$
$3 x=51^{\circ}$
$5 x+2^{\circ}=5 \times 17^{\circ}+2=87^{\circ}$
29. (C) Area of $\triangle C D E=\frac{1}{2} \times C D \times A D$
$=\frac{1}{2} \times 17.8 \mathrm{~cm} \times 11.7 \mathrm{~cm}$
$=104.13 \mathrm{~cm}^{2}$
$\therefore$ Area of the shaded region $=104.13 \mathrm{~cm}^{2}$ (or)

Area of the rectangle - area of $\triangle C D E$
$=17.8 \times 11.7 \mathrm{~cm}^{2}-104.13 \mathrm{~cm}^{2}$
$=208.26 \mathrm{~cm}^{2}-104.13 \mathrm{~cm}^{2}$
$=104.13 \mathrm{~cm}^{2}$
30. (B) $\ln \triangle B C D, B D=C D$

$\angle B C D=\angle C B D=a$
$\therefore a+a+110^{\circ}=180^{\circ}$
$2 \mathrm{a}=180^{\circ}-110^{\circ}=70^{\circ}$
$a=\frac{70}{2}=35^{\circ}$
In $\triangle A B D, A D=B D$
$\angle \mathrm{ABD}=\angle \mathrm{BAD}=x$
In $\triangle A C D, A D=C D$
$\angle \mathrm{ACD}=\angle \mathrm{CAD}=y$
In $\triangle A B C$,
$\angle \mathrm{DAB}+\angle \mathrm{BAD}+\angle \mathrm{DAC}+\angle \mathrm{ACD}+\angle \mathrm{BCD}$
$+\angle \mathrm{CBD}=180^{\circ}$
$x+x+y+y+35^{\circ}+35^{\circ}=180^{\circ}$
$2 x+2 y=180^{\circ}-70^{\circ}=110^{\circ}$
$2(x+y)=110^{\circ}$
$x+y=\frac{110^{\circ}}{2}=55^{\circ}$
$\therefore \angle \mathrm{BAC}=x+y=55^{\circ}$

## MATHEMATICS - 2

31. (B, C)

An isosceles triangle has one line of symmetry A rectangle has two lines of symmetry
A square has four lines of symmetry
A trapezium has one line of symmetry.
32. (A, B, C)

Option $A$ it is in the form of $p / q$ where $p, q \varepsilon z \&$ $\mathrm{q}=0$
$\therefore \frac{22}{7}$ is a rational number.
Option B: $1 . \overline{56}$ is non terminating but repeating decimal
$\therefore 1 . \overline{56}$ is a rational number
Option C: 1.565675678 is a terminating decimal
$\therefore$ It is a rational number
Option D: It is neither terminates nor repeats
$\therefore$ It is not a rational number
33. (A, C)
$\Delta \mathrm{AFB} \cong \Delta \mathrm{EFD} \quad[\because$ SAS congruence's $]$


In $\triangle$ BCF and $\triangle \mathrm{DCF}$
$B C=C D C$ (side)
$\angle \mathrm{FCB}=\angle \mathrm{DCF}=\angle \frac{\mathrm{BCD}}{2}$
$F C=F C \quad(\because$ Common side $)$
$\therefore \triangle \mathrm{BCF} \cong \triangle \mathrm{DCF}[\because$ SAS congruence's]
34. (A, B, C, D)

All options are true for drawing the parallel lines.
35. (A, B, C, D)

Option A: Given $3^{x}=270$
$\therefore 3^{x-2}=\frac{3^{x}}{3^{2}}$
$3^{x-2}=\frac{270}{9}$
$\therefore$ Option A true

Option B: $\left(\frac{x^{a}}{x^{b}}\right)^{c} \times\left(\frac{x^{b}}{x^{c}}\right)^{a} \times\left(\frac{x^{c}}{x^{a}}\right)^{b}$
$=\frac{x^{\mathrm{ac}}}{x^{\mathrm{bc}}} \times \frac{x^{\mathrm{ab}}}{x^{\mathrm{ac}}} \times \frac{x^{\mathrm{bc}}}{x^{\mathrm{ab}}}=1$
$\therefore$ Option B true

Option C : $\left(\frac{\mathrm{p}}{\mathrm{q}}\right)=\frac{\left(\frac{2}{3}\right)^{3}}{\left(\frac{3}{2}\right)^{-3}}=\left(\frac{2}{3}\right)^{3} \times\left(\frac{3}{2}\right)^{3}$
$=\left(\frac{2}{3} \times \frac{3}{2}\right)^{3}$
$=1^{3}$
$\therefore \frac{p}{q}=1$
$\therefore\left(\frac{p}{q}\right)^{-10}=\frac{1}{\left(\frac{p}{q}\right)^{10}}=\frac{1}{\left(1^{10}\right)}=1$
$\therefore$ Option C is true
Option D: Given $2^{2} \times\left(2^{3}\right) \mathrm{m}=2^{-7}$
$2^{2} \times 2^{3 m}=2^{-7}$
$\therefore 2^{3 m+2}=2^{-7}$
$\therefore 3 m+2=-7$
$3 m=-7-2$
$3 m=-9$
$m=-\frac{9}{3}=-3$

## REASONING

36. (D) Every time one letter is coded as number from starting position and ending position alternatively.

CHAPTER, 1 HAPTER , CHAPTE2, C3APTER , CHAPTIR , CH5PTER , CHAPGER
37. (D)

38. (D) If $15^{\text {th }}$ August 2022 is Monday, then $15^{\text {th }}$ August 2023 is Tuesday and $15^{\text {th }}$ August 2024 is Thursday(2024 is a leap year)
$16^{\text {th }}$ August 2024——Friday
$17^{\text {th }}$ August 2024——Saturday
$18^{\text {th }}$ August 2024——Sunday
19 ${ }^{\text {th }}$ August 2024——Monday
39. (A) The square increases in size and goes in the middle, the circle increases in size and goes at the bottom and the rectangle rotates $90^{\circ}$ and goes at the top.
40. (A) The answer is TARIC

| Letter | B | E | L | T | S | U | M |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Code | M | R | I | C | T | B | A |

41. (C) 7 and 8. Any two numbers are as likely as the rest, as the difference between the numbers in the answer options is always one.
42. (A)

43. (D) $A=0, B=1, C=2, D=3, E=4, F=5$, $G=6, H=7, I=8, J=9$
Option (A) 1309 = BDOJ ('O' vowel)
Option (B) 7432 = HEDC ('E' vowel)
Option (C) $6278=\mathrm{GCHI}$ ('I' vowel)
Option (D) 2516 = CFBG
(All letters are consonants)
44. (C) It is given that, Sandeep was facing the pizza hut at the beginning and he turned anti-clockwise to face south-east, then

From the figure, It is clearly shown that the pizza hut is in WEST and moving anticlockwise from West to South-East. $135^{\circ}$ angle is formed
Hence, option (C) is correct

45. (A) It is clear than that $F, E, A, B, G, H, C$ and $D$ is a possible order. It is a possible that ' $E$ ' finished before $A$ and it is possible that G and H finish before C . The rest of the ordering is mandated by the constrains.

## CRITICAL THINKING

46. (B) 4 lines are required to touch each circles without lifting the pen.

47. (C) If Rohit always tells the truth, then both Honey and Rohit speak Tamil (Statements I and II) and Honey is lying (Statement III)
48. (A) As shown in the picture below, you should tilt the barrel so the liquid will touch the upper end of the barrel. So if the bottom part of the barrel is visible then it's not half full.

49. (A) Since large numbers of primary schools in the rural areas are run by only one teacher so there has been a huge dropout from the primary schools in rural areas.
50. (B) Total 7 ways
$4 \times 7=28$ ways
$\frac{1}{4}{ }^{\text {th }}$ part of a diagram is


The End

